

↳ Conditional Proposition: if P and Q are propositions, the compound proposition "if P then Q " denoted by $P \rightarrow Q$ is called a conditional proposition or implication and the connective is the conditional connective. The proposition P is called antecedent or hypothesis, and the proposition Q is called the consequent or conclusion.

Example 1: P : Tomorrow is Sunday \rightarrow
 Q : Today is Saturday

$P \rightarrow Q$: if tomorrow is Sunday then today is Saturday
 hypothesis/antecedent conclusion/consequent

example 2: P : I get the increment
 Q : I will give the party

$P \rightarrow Q$: if I get the increment then I will give the party.
 hypothesis conclusion.

Truth Table

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
f	F	T

Calculate the truth table for

i) $P \vee \neg P$

ii) $(P \vee \neg P) \rightarrow P$

i) $P \vee \neg P$:

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

ii) $(P \vee \neg P) \rightarrow P$:

$P \vee \neg P$	P	$(P \vee \neg P) \rightarrow P$
T	T	T
T	F	F

Logical Equivalence / Logical Equivalent:

if two proposition $P(p, q, \dots)$ and $Q(p, q, \dots)$ where p, q, \dots are propositional variable have the same truth values in every possible case, the proposition are called logically equivalent, and denoted by $P(p, q, \dots) \equiv Q(p, q, \dots)$.

Q) use truth table show that

$$P \rightarrow Q \equiv \neg P \vee Q$$

P	Q	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Same truth values for all input combinations.

So, $\neg P \vee Q \equiv P \rightarrow Q$

Q) use truth table show that

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Same truth values for all input combinations.

So, $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$.

Negation of Implication:

as we know that $P \rightarrow Q \equiv \neg P \vee Q$

$$\text{So, } \neg(P \rightarrow Q) \equiv \neg(\neg P \vee Q)$$

$$\equiv \neg(\neg P) \wedge \neg Q$$

$$\equiv P \wedge \neg Q$$

$$\neg(P \rightarrow Q) \equiv P \wedge \neg Q$$

Example: if sky is blue then milk is white

P: sky is blue

Q: milk is white

$P \rightarrow Q$

its negation is

sky is blue and milk is not white.

a) write the negation of each of the following conditional statements.

a) if the door is unlocked, the alarm sounds.

→ The door is unlocked, and the alarm doesn't sound.

b) if it is raining, then the game is cancelled.

→ it is raining and the game is not cancelled.

c) if he studies, he will pass the examination

→ he studies and he will not pass the examination.